Second order systems

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Trial lecture at the Department of Electrical Engineering and Computer Science,
University of Stavanger, Norway, 2017

About Me

Education:

- PhD in Engineering Cybernetics, Norwegian University of Science and Technology (NTNU), Norway
- MSc in Computer Science Engineering, University of Siena, Italy
- BSc degree in Computer Science Engineering, University of Catania, Italy

Mobility:

- Visiting Fellow, Technical Aspects of Multimodal Systems (TAMS), Department of Mathematics, Informatics and Natural Sciences, University of Hamburg, Hamburg, Germany
- Visiting Student, School of Computing and Intelligent Systems, University of Ulster, Londonderry, United Kingdom
- Granted with an Erasmus+ Staff Mobility for Teaching and Training project

Activities:

- Membership Development Officer for the IEEE Norway Section

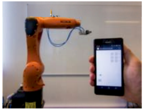
Research topics

Visualisation/Game Development

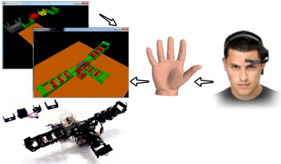
Augmented Reality/Virtual Reality

Micro-controllers, IoT, Maker Tech.

Mobile Device

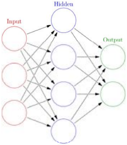


Software/Hardware Codesign



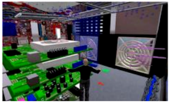
Real-time Systems

Artificial Intelligence



Safety-Critical Systems

Education



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About Me

Current position:

- Filippo Sanfilippo, Postdoctoral Fellow at the Dept. of Eng. Cybernetics, NTNU, Trondheim, Norway

Current courses:

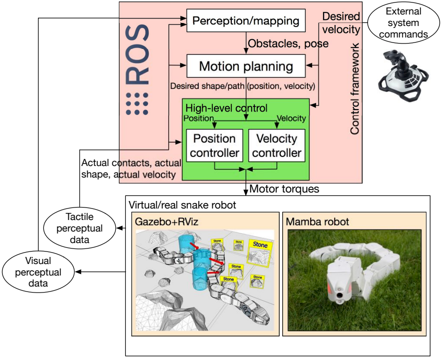
- TTK4235 - Embedded Systems (Lecturer)
- Experts in Teamwork - Snake robots (Supervisor)

Past courses:

- Real-time Computer Programming (Lecturer)
- Mechatronics, Robots and Deck Machines (Teaching Assistant)
- System Simulation in Matlab/Simulink (Lecturer)

Current research topic:

- "SNAKE - Control Strategies for Snake Robot Locomotion in Challenging Outdoor Environments", project number 240072, supported by the Research Council of Norway through the *Young research talents funding scheme*



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First-order linear systems

Canonical homogeneous first-order differential equation:

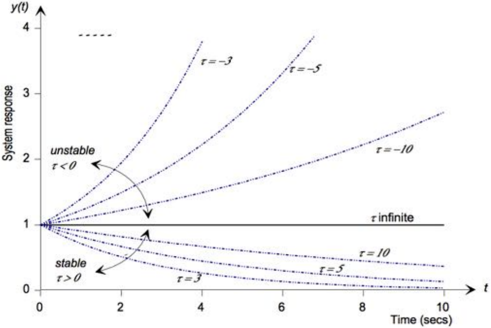
$$\tau \frac{dy(t)}{dt} + y(t) = f(t), \qquad (1)$$

where $\tau \neq 0$ is the system *time constant*. The characteristic equation is given by:

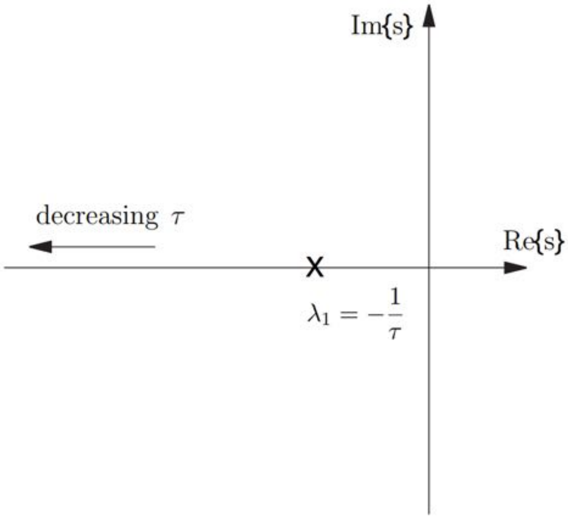
$$\tau\lambda + 1 = 0, \qquad (2)$$

which has a single root, $\lambda = -1/\tau$. The system response to an initial condition $y(0)$ is:

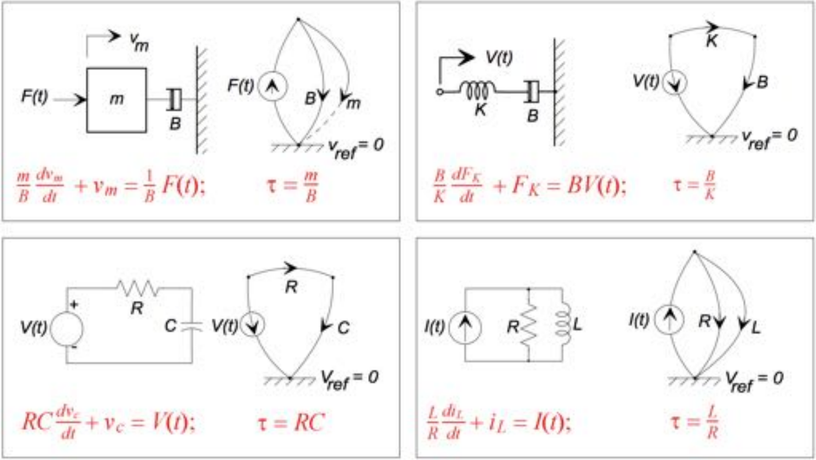
$$y_h(t) = y(0)e^{\lambda t} = y(0)e^{-t/\tau}, \qquad (3)$$



First-order linear systems: s-plane



First-order linear systems



The characteristic response of first-order systems

- The canonical homogeneous first-order differential equation is given by:

$$\tau \frac{dy(t)}{dt} + y(t) = f(t). \tag{4}$$

The solution (given $f(t)$ and $y(0) = 0$) is the *characteristic first-order response*.

- The first-order homogeneous solution is of the form of an exponential function:

$$y_h(t) = e^{-\lambda t}, \lambda = 1/\tau. \tag{5}$$

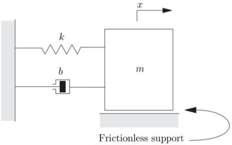
- The total response $y(t)$ is the sum of two components:

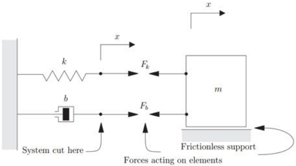
$$y(t) = y_h(t) + y_p(t) = ce^{-t/\tau} + y_p(t), \tag{6}$$

where c is a constant to be found from the initial condition $y(0) = 0$, and $y_p(t)$ is a particular solution for the given forcing function $f(t)$.

Input $u(t)$	Characteristic Response	Input/Output Response $y(t)$ for $t \geq 0$
$u(t) = 0$		$y(t) = y(0)e^{-t/\tau}$
$u(t) = u_r(t)$	$y_r(t) = t - \tau\left(1 - e^{-t/\tau}\right)$	$y(t) = \left[q_0 t + (q_1 - q_0\tau)\left(1 - e^{-t/\tau}\right)\right]$
$u(t) = u_s(t)$	$y_s(t) = y_s(t) = 1 - e^{-t/\tau}$	$y(t) = \left[q_0 - \left(q_0 - \frac{q_1}{\tau}\right)e^{-t/\tau}\right]$
$u(t) = \delta(t)$	$y_\delta(t) = \frac{1}{\tau}e^{-t/\tau}$	$y(t) = \frac{q_1}{\tau}\delta(t) + \left(\frac{q_0}{\tau} - \frac{q_1}{\tau^2}\right)e^{-t/\tau}$

Second-order linear systems





$$-F_b - F_k = -b\frac{dx}{dt} - kx = m\frac{d^2x}{dt} \Rightarrow m\frac{d^2x}{dt} + b\frac{dx}{dt} + kx = 0. \tag{7}$$

- $x(0) = x_0, v(0) = \dot{x}(0) = v_0$ are required initial conditions
- assume that $x(t)$ takes the form $x(t) = ce^{st}$

$$ms^2ce^{st} + bsce^{st} + kce^{st} = 0 \Rightarrow ms^2 + bs + k = 0. \tag{8}$$

$$s_1 = -\frac{b}{2m} + \frac{\sqrt{b^2 - 4mk}}{2m}, s_2 = -\frac{b}{2m} - \frac{\sqrt{b^2 - 4mk}}{2m}. \tag{9}$$

- s_1, s_2 are the pole locations (natural frequencies) of the system.
- In most cases $b2 \neq 4mk$) and the initial condition response will take the form:

$$x(t) = c_1e^{s_1t} + c_2e^{s_2t}. \tag{10}$$

[1]

[1] MIT OpenCourseWare. *Review of complex numbers*. 2017. URL: https://goo.gl/utPL5y.

Standard terms

The pole locations are parameterized in terms of the natural frequency ω_n, and the damping ratio ζ where:

$$\omega_n = \sqrt{\frac{k}{m}}, \tag{11}$$

$$\zeta = \frac{b}{2\sqrt{km}}. \tag{12}$$

Natual frequency and damping ratio:

- The natural frequency, ω_n, is the frequency at which the system would oscillate if the damping, b, were zero
- The damping ratio, ζ, is the ratio of the actual damping, b, to the critical damping, $b_c = 2\sqrt{km}$

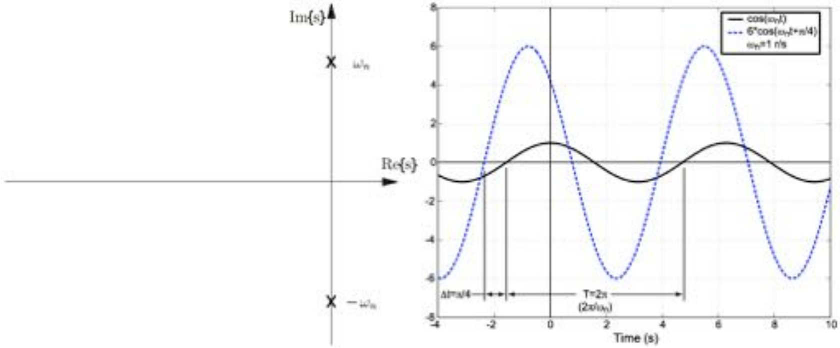
$$m\frac{d^2x}{dt} + b\frac{dx}{dt} + kx = 0 \Rightarrow \frac{1}{\omega_n^2}\frac{d^2x}{dt} + \frac{2\zeta}{\omega_n}\frac{dx}{dt} + x = 0. \tag{13}$$

Physically reasonable assumption: the values of m, and k are greater than zero (to maintain system order) and that b is non-negative (to keep things stable).

Pole locations

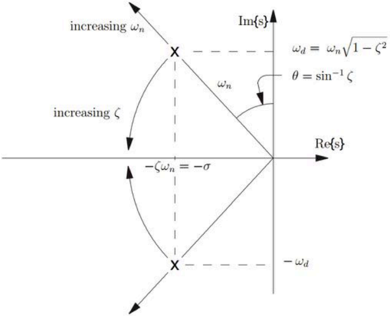
- If $b = 0$, the poles are complex conjugates on the imaginary axis at $s_1 = +j\sqrt{k/m}, s_2 = -j\sqrt{k/m}$. This corresponds to $\zeta = 0$, and is referred to as the undamped case
- If $b^2 - 4mk < 0$ then the poles are complex conjugates lying in the left half of the s-plane. This corresponds to the range $0 < \zeta < 1$, and is referred to as the underdamped case
- If $b^2 - 4mk = 0$ then the poles coincide on the real axis at $s_1 = s_2 = -b/2m$. This corresponds to $\zeta = 1$, and is referred to as the critically damped case
- If $b^2 - 4mk > 0$ then the poles are at distinct locations on the real axis in the left half of the s-plane. This corresponds to $\zeta > 1$, and is referred to as the overdamped case

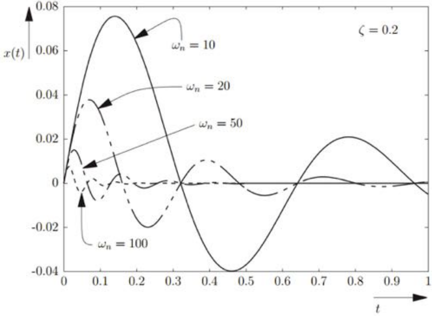
Undamped case ($\zeta = 0$)



The poles lie at $s_1 = j\omega_n$ and $s_2 = -j\omega_n$.

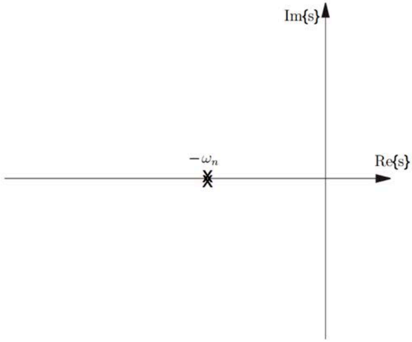
Underdamped case $(0 < \zeta < 1)$

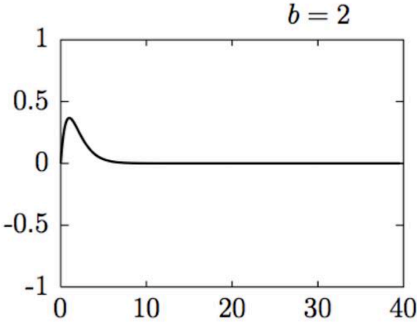




The poles lie at $s = -\sigma \pm j\omega_d$, where $\sigma = \zeta\omega_n$ is the attenuation, and $\omega_d = \omega_n\sqrt{1-\zeta^2}$.

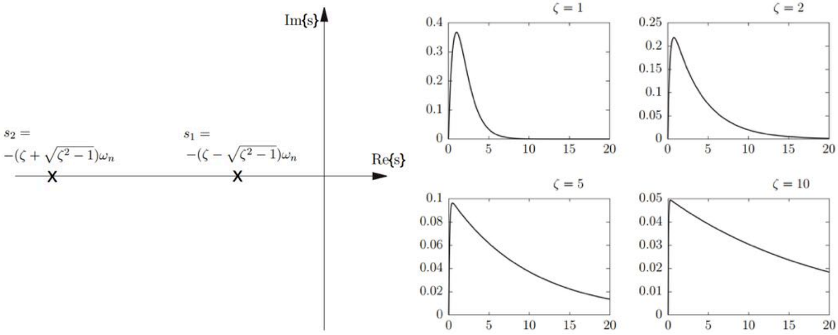
Critically-damped case ($\zeta = 1$)





The poles lie at $s_1 = s_2 = -\omega_n$.

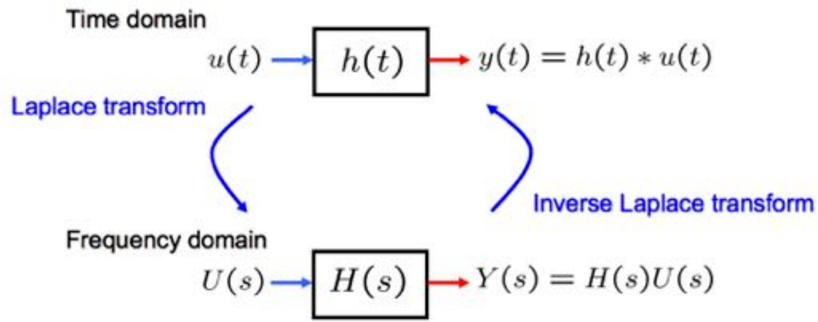
Overdamped case ($\zeta = 1$)



The poles lie at $s_1 = -(\zeta - \sqrt{\zeta^2 - 1})\omega_n$ and $s_2 = -(\zeta + \sqrt{\zeta^2 - 1})\omega_n$.

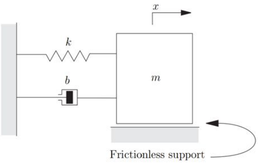
Laplace transforms and transfer functions

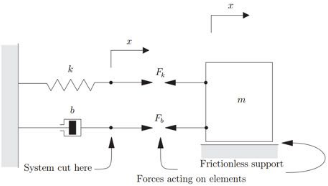
Laplace transforms: method for solving differential equations, converts differential equations in time t into algebraic equations in complex variable s.



- Defined as the ratio of the Laplace transform of the output signal to that of the input signal (think of it as a gain factor!)
- Contains information about dynamics of a Linear Time Invariant system

Mass-spring-damper system





ODE

$$M\ddot{y}(t) + b\dot{y}(t) + ky(t) = u(t)$$

Assume all initial conditions are zero. Then take Laplace transform,

$$Ms^2Y(s) + bsY(s) + kY(s) = U(s)$$

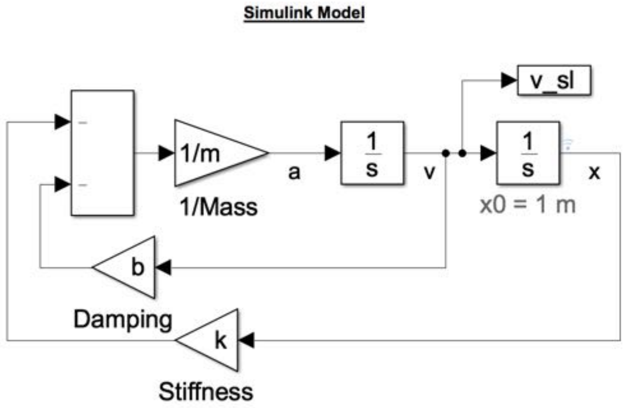
Output

$$\boxed{\frac{Y(s)}{U(s)}} = \boxed{\frac{1}{Ms^2 + bs + k}}$$

Transfer function

Input

Mass-Spring-Damper in Simulink

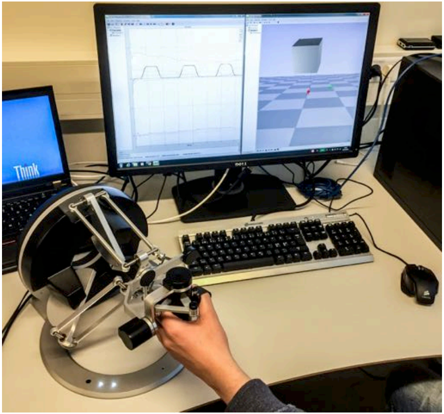


$$m\frac{d^2x}{dt} + b\frac{dx}{dt} + kx = 0. \tag{14}$$

[2]

[2] MathWorks. *Mass-Spring-Damper in Simulink and Simscape*. 2017. URL: https://goo.gl/Ftr6hK.

Applications



[3]

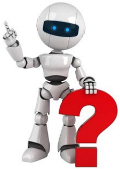
[3] Filippo Sanfilippo, Paul B.T. Weustink, and Kristin Ytterstad Pettersen. "A coupling library for the force dimension haptic devices and the 20-sim modelling and simulation environment". In: *Proc. of the 41st Annual Conference of the IEEE Industrial Electronics Society (IECON), Yokohama, Japan*. 2015, pp. 168–173.

Conclusion

Learning experience:

- First-order linear systems and characteristic response
- Second-order linear systems and characteristic response
- Several applications

Thank you for your attention



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[4] Filippo Sanfilippo et al. "Virtual functional segmentation of snake robots for perception-driven obstacle-aided locomotion". In: *Proc. of the IEEE Conference on Robotics and Biomimetics (ROBIO), Qingdao, China.* 2016, pp. 1845–1851.

[5] Filippo Sanfilippo et al. "A review on perception-driven obstacle-aided locomotion for snake robots". In: *Proc. of the 14th International Conference on Control, Automation, Robotics and Vision (ICARCV), Phuket, Thailand.* 2016, pp. 1–7.

[6] Filippo Sanfilippo et al. "Perception-driven obstacle-aided locomotion for snake robots: the state of the art, challenges and possibilities". In: *Applied Sciences* 7.4 (2017), p. 336.

References

[1] MIT OpenCourseWare. *Review of complex numbers*. 2017. URL: https://goo.gl/utPL5y.

[2] MathWorks. *Mass-Spring-Damper in Simulink and Simscape*. 2017. URL: https://goo.gl/Ftr6hK.

[3] Filippo Sanfilippo, Paul B.T. Weustink, and Kristin Ytterstad Pettersen. "A coupling library for the force dimension haptic devices and the 20-sim modelling and simulation environment". In: *Proc. of the 41st Annual Conference of the IEEE Industrial Electronics Society (IECON), Yokohama, Japan*. 2015, pp. 168–173.

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