Modelling and Control of Continuum Manipulators

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Trial Lecture
Summary

1. Introduction
2. Design Principles and Challenges
3. Kinematics
4. Forces, Dynamics, and Control
5. Ongoing Research and Analogies with Synergistic Modular Grasping
6. Conclusions
Traditional Robotic Manipulators

What is a *Manipulator*?

- In robotics, a device used to manipulate materials without direct contact.
- An *arm-like* or *hand-like* mechanism with RIGID LINKS.
- In industrial ergonomics, a *lift assist device*.
Advantages and limitations of traditional *rigid-link* manipulators:

- Excellent for precise positioning of their end-effector.
- Outside the highly structured world of industry, they are less successful. Their rigid-link structure tends to be the cause of unwanted collisions.
- Inability to grasp objects other than at their end-effector.

Alternative design: robots with a *serpentine* or *continuous* form\(^1\).

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Rigid-Link (Discrete) Vs. Serpentine Vs. Continuum Manipulators

Serpentine Manipulators:
- use discrete joints but combine very short rigid links with a large density of joints. Similar to a snake.

Continuum Manipulators:
- do not contain rigid links. The structures bend continuously. Similar to the tentacles or tongues.
Continuum Manipulators

Continuous Backbone Robot Manipulators, also known as Continuum Manipulators:

- “invertebrate” Vs. “vertebrate” robots;
- hyper-redundant and compliant design;
- no prior planning.
Challenges Concerning Continuum Manipulators

Design challenges:
- a continuous backbone, whose shape can be actuated in some way;
- compliant backbone that allows for smoothly adapt to externally applied loads.

Control challenges:
- the relationship between the shapes and inputs is highly complex.
Not Necessarily Continuum Designs

Muscular hydrostats are structures comprised almost entirely of their own actuators (muscle), with some additional fluid and connective tissue:

- octopus arms, elephant trunks, squid tentacles, and mammalian tongues, ...
- animals do not have to be the only source of inspiration; the vines and tendrils of plants.
Some works aimed to mimic the muscular hydrostat design concept have been presented. But this technology is still not mature.

Intrinsic Vs. Extrinsic actuation.

Three alternative fundamental design strategies have emerged:

- Tendon-Based Designs;
- Concentric Tube Designs;
- Locally Actuated Backbone Designs.

Tendon-Based Designs

- Most direct approach: use of remotely actuated tendons.
- Given a backbone which, tendons can be used to deviate it from a given shape.
- Forces applied to the tendons at the base produce torques.

[4,5]


Tendon-Based Designs

One choice for the core backbone element is a compressible spring:

+ natural compliance;

− difficult to control, effort intended for backbone bending is lost in compression\(^6\).

Simple solution: flexible incompressible rod as the backbone element\(^7\). Slender low-profile backbone and more predictable behavior but also preclusion of backbone extension.

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Tendon-based continuum designs share the following general features:

- finite series of “sections”;
- relatively high forces but slack and backlash issues;
- relatively bulky actuator (extrinsically actuated).

Tendon-actuated continuum robots have been designed for space operations and in some medical procedures[8].

Concentric Tube Designs

- Extrinsically actuated continuum robot (and the most recent to emerge).
- Based on a backbone formed by concentric compliant tubes.
- The tubes are free to move (translate and rotate) with respect to each other (telescope)[9].

However, it does not inherently provide for backbone bending:

- precurved compliant tubes;
- tendons to bend the tubes.

Concentric Tube Designs

Advantages and disadvantages:

+ inherently clean and thin design. Smaller-scale and lower-force device. Application in the medical field\(^{[10]}\). Sometimes termed “active cannulas”;
+ actuator values (unlike with tendons) directly correspond to backbone shape variables;
– need for an external actuator package and the lack of inherent support for actively controlled bending.

Locally Actuated Backbone Designs

- Intrinsically actuated continuum robot (closest design to the biological continuum structures).
- Typically three independently actuated muscles that can be “extenders” or “contractors”.

+ Key advantage of inherently providing the backbone with extension, bending, and torsion.

− Low-force generation capabilities, fairly complex tube routing/valving, and the need for external pressure regulation equipment and a compressor.

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Locally Actuated Backbone Designs

Variable Stiffness Continuum Robot Design

Common Property: Constant Curvature

Common properties:
- serially connected sections;
- the internal potential energy in each section is uniformly distributed.

Constant Curvature:
- the resulting backbone approximates a serially connected set of “constant curvature” sections.
- the “constant curvature” property is affected by external loading but it remains a good first approximation.
Rigid-Link Vs. Continuum Robots

**Rigid-Link Robots:**
- the Denavit-Hartenberg (D-H) convention as a framework for the development of kinematic (and dynamic) models.
- Local coordinate frame fixed in each of the (finite number of) links and a sequential series of frame-to-frame steps.

**Continuum Robots:**
- the local shape varies continuously along the backbone.

**Main approaches:**
- a “bottom-up” strategy, exploiting the D-H approach to fit a “virtual” rigid-link robot;
- modal approaches.
A “Virtual” Three-Joint Rigid-Link Manipulator

Three discrete transformations:

- a rotation to “point” the tangent at the curve beginning to the curve end point;
- a translation along the newly aligned direction;
- a second rotation (of same amount as the first) to realign with the tangent at the curve’s end.

A “Virtual” Three-Joint Rigid-Link Manipulator

Note: \( \theta_1 = \theta_3 = \theta \), \( d_2 = \|x(s)\| \).

\[
\begin{align*}
\frac{i-1}{i} T &= \begin{bmatrix}
    c \theta_i & -s \theta_i & 0 & a_{i-1} \\
    s \theta_i c \alpha_{i-1} & c \theta_i c \alpha_{i-1} & -s \alpha_{i-1} & -s \alpha_{i-1} d_i \\
    s \theta_i s \alpha_{i-1} & c \theta_i s \alpha_{i-1} & c \alpha_{i-1} & c \alpha_{i-1} d_i \\
    0 & 0 & 0 & 1
\end{bmatrix} \\
0_3 T &= 0_1 T_2 T_3 = \begin{bmatrix}
    c(\theta_1 + \theta_3) & -s(\theta_1 + \theta_3) & 0 & -d_2 s \theta_1 \\
    s(\theta_1 + \theta_3) & c(\theta_1 + \theta_3) & 0 & d_2 c \theta_1 \\
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1
\end{bmatrix}
\end{align*}
\]

Table: D-H Table of a “Virtual” Three-Joint Rigid-Link Manipulator

<table>
<thead>
<tr>
<th>Link</th>
<th>( \theta )</th>
<th>( d )</th>
<th>( a )</th>
<th>( \alpha )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>*</td>
<td>0</td>
<td>0</td>
<td>-90</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>*</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>3</td>
<td>*</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
A “Virtual” Three-Joint Rigid-Link Manipulator

Utilising the underlying geometry, the arch length \( s \) can be obtained as:

\[
s = r(2\theta) = \frac{(2\theta)}{k} = \frac{(\theta_1 + \theta_3)}{k}.
\]

Therefore:

\[
(\theta_1 + \theta_3) = sk.
\]

Also:

\[
\frac{\|x(s)\|}{2} = \frac{d_2}{2} = rsin\theta = \frac{sin\theta}{k}.
\]

Therefore:

\[
d_2 = \frac{2sin\theta}{k}.
\]

Note: \( \theta_1 = \theta_3 = \theta \), \( d_2 = ||x(s)|| \). Denoting the curvature with \( k \) and the radius with \( r \) we get:

\[
k = \frac{1}{r}.
\]

\[
0_3 \mathbf{T} = \begin{bmatrix}
  c(\theta_1 + \theta_3) & -s(\theta_1 + \theta_3) & 0 & -d_2s\theta_1 \\
  s(\theta_1 + \theta_3) & c(\theta_1 + \theta_3) & 0 & d_2c\theta_1 \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix} = \begin{bmatrix}
  c(sk) & -s(sk) & 0 & \frac{1}{k}[c(sk) - 1] \\
  s(sk) & c(sk) & 0 & \frac{1}{k}s(sk) \\
  0 & 0 & 1 & 0 \\
  0 & 0 & 0 & 1
\end{bmatrix}.
\]

Planar multi-section kinematic models can be easily created by chaining together the models for the individual sections.
A “Virtual” Five-Joint Rigid-Link Manipulator

- The kinematics of spatial constant curvature curves can similarly be modeled by the addition of an extra pair of (again identical, coupled) rotations to each end of the planar version to create a 3D virtual rigid-link robot.
- Multi-section 3D kinematic models can be created by chaining together individual section models.

**Table: D-H Table of a “Virtual” Five-Joint Rigid-Link Manipulator**

<table>
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<td>-90</td>
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<tr>
<td>3</td>
<td>0</td>
<td>*</td>
<td>0</td>
<td>90</td>
</tr>
<tr>
<td>4</td>
<td>*</td>
<td>0</td>
<td>0</td>
<td>-90</td>
</tr>
</tbody>
</table>
The previous approach is fairly complex:

- an alternative strategy is to “build” backbone shapes via a finite number of simple modal functions.

\[ k(s) = \sum_{i=1}^{n} \mu_i \phi_i(s), \]

where \( \mu_i \) are coefficients, and \( \phi_i(s) \) are the modal functions. The coefficients become the “configuration” of the robot.

- The use of the classical trigonometric basis functions appears a natural choice for the modal functions.
- However, an infinite number of trigonometric modes are needed.

Using Modal Functions

In [17], (the first few elements of) two alternative sets of Wavelet basis functions are used. These use the “natural basis set” or “box functions”:

\[ k(s) = \sum_{i=1}^{n} \mu_i \phi_i^b(s), \]

and the “Haar” basis set:

\[ k(s) = \sum_{i=1}^{n} \mu_i \phi_i^w(s). \]

Key Advantages:

- the robot shape can be parameterised by a finite set of user-selected functions with convenient properties;
- the number of “modes” used can be user-selected, for example, to constrain the computational complexity of the resulting model;
- eliminate the singularities.

Using Modal Functions

Air-OCTOR has a single (extensible) section with circular cross-section, actuated by three tendons, spaced at 120 degrees\cite{18}.

The three tendons are routed through a series of $n - 1$ intermediate connection points before being terminated at the end of the section. During actuation, this causes the tendons to form $n$ straight line segments within the section.

The length $h_c$ of a (imaginary) tendon running directly through the center of a single such segment of the section is given by:

$$h_c = \frac{l_3 + l_2 - 2l_1}{6n},$$

where the shortest tendon length is $l_1$, and $n$ is the number of segments in the section.
By applying a projection onto the \((z, \phi)\) plane, the expressions for the curvature, \(k_\phi\), and angle of curvature, \(\phi\), in terms of tendon lengths can be obtained:

\[
k_\phi = 2 \frac{\sqrt{l_1^2 + l_2^2 + l_3^2 - l_1 l_2 - l_2 l_3 - l_1 l_3}}{d(l_1 + l_2 + l_3)},
\]

\[
\phi = \arctan\left(\frac{\sqrt{3} l_3 + l_2 - 2 l_1}{l_2 - l_3}\right),
\]

where \(d\) is the radius of the section cross-section. Finally, after some further geometrical analysis, it can be shown that:

\[
s = \frac{nd(l_1 + l_2 + l_3)}{\sqrt{l_1^2 + l_2^2 + l_3^2 - l_1 l_2 - l_2 l_3 - l_1 l_3} \arcsin\left(\frac{\sqrt{l_1^2 + l_2^2 + l_3^2 - l_1 l_2 - l_2 l_3 - l_1 l_3}}{3nd}\right)}.
\]

Figure: View from side of one segment of section.
Conventional techniques for formulating Jacobians for rigid-link robots can be applied to “virtual” rigid-link manipulators to derive a “continuum” Jacobian\[^{[19]}\].

- Any of the kinematic relationships (modal or direct) can be differentiated to find the appropriate Jacobian.
- A Jacobian (pseudo-) inverse can then be used to iteratively solve configuration space rates given desired tip rates:

\[
\frac{dx(s)}{dt} = [J(\mu, s)] \frac{d\mu}{dt}.
\]

Using Continuum Robots for Locomotion and for Grasping:

- significant deviation from constant curvature;
- the deviation is also a desired property.

[20,21]


Forces and External Loading

[21]
Approaches based on the well-understood Lagrangian\cite{22} and Newton-Euler\cite{23} methods have been established.

**The Lagrangian dynamics approach is outlined:**

- there are several steps which are specialized to continuum robot scenario.

**Key steps:**

- model the backbone as being comprised of circular cross-sectional “slices” of infinitesimal thickness;

- each slice, at a location $\sigma$ along the backbone, has mass $m(\sigma)$, inertia tensor $I(\sigma)$, and first moment of inertia $m(\sigma)r(\sigma)$, where $r(\sigma)$ is the distance from the slice geometric center to its center of mass.

\cite{22} Isuru S Godage et al. “Shape function-based kinematics and dynamics for variable length continuum robotic arms”. In: *Proc. of the IEEE International Conference on Robotics and Automation (ICRA)*. 2011, pp. 452–457.

Dynamics

**Strategy:**
- kinetic and potential energy of each slice;
- total energies $K$ and $P$ (via integration along the backbone);
- substitute $L = K - P$ into Lagrange’s equations to find the dynamic model:

\[
\frac{d}{dt} \left[ \frac{\partial L}{\partial \dot{\theta}_i(\sigma, t)} \right] - \frac{\partial L}{\partial \theta_i(\sigma, t)} = \tau_i(\sigma, t),
\]

\[
i = 1, \ldots, n.
\]

$\theta_i$ correspond to the $n$ actuated configuration space variables, $\tau_i$ are the corresponding forces.

- After forming $L = K - P$ and substituting into Lagrange’s equations, the resulting dynamic model takes the form of:

\[
M \ddot{q} + V(q, \dot{q}) \dot{q} + g(q) = \tau.
\]

- It matches the dynamics of rigid-link robots, apart from being continuous in nature.

- The inertia matrix $M$ can be shown to be positive definite and symmetric, and it satisfies the property (useful for control):

\[
\xi^T (\dot{M} - 2V) \xi = 0, \forall \xi \in \mathbb{R}^n.
\]

(skew-symmetric property) used to cancel the non-linearities of the Coriolis matrix from Lyapunov functions.
Newton-Euler approaches:
- computationally demanding

Lumped-parameter models, based on linear mass-spring-damper elements:
- the model is tuned to octopus-inspired underwater operation and includes terms to model buoyancy and drag;
- trading off computational complexity of the model against accuracy.

Control

- At which level to close the loop.
- Sensed quantities are usually limited.
- For example, two closed-loop control implementations applied to a small scale continuum manipulator.

Control Examples

[25]
The modular approach\textsuperscript{[26]} allows for realising a “Serpentine” structure.

Synergystic control approach\textsuperscript{[27,28]} (imaginary software cables/tendons).


Analogies with Synergistic Modular Grasping
Analogies with Synergistic Modular Grasping

State of the Art Review:

- Three fundamentally different designs.
- Kinematics of continuum robots has reached a mature stage, with theory matching most of the corresponding results for rigid-link robots.
  - However, continuum robots present issues and difficulties not present for rigid-link robots, due to the inherent compliance and infinite DOFs.
- Models which take into account the effects on the kinematics from external loading have been established.
- Synergy-based methods.

Future Challenges:

- Huge potential for applications in several fields including grasping, terrain-adapting continuum-limbed vehicles, ship-to-ship refueling, and exploration of extraterrestrial surfaces.
Thank you for your attention


